Note on the Put-Call Parity Equation

The put-call parity equation encapsulates the inverse relationship between the benefits and costs associated with a call, as opposed to, a put option contract. Put-call parity defines the relationship between the price of a European call option and a European put option contracts with identical strikes on the same expiration date. Put-call parity specifies that a portfolio consisting of a long call [against increases in the price of the underlying] plus a short put option [against a decrease in the price of the underlying security] is equal to a forward contract for the underlying at the strike price at the time of expiration.

The put-call parity relationship represents a static model and it is assumed that transaction and financing costs will be negligible. In practice, even with these heroic assumptions, the liquidity of the options market allows the results of the put-call equation to approximate the pricing of the call and put options.

According to the put-call parity equation:

 C - P = PV[ FS -X]

The difference between the current call and put premiums [C -P] is equal to the present value [PV]of the gain or loss in the underlying stock relative to the strike [ FS: future stock price less the strike X].

Left Hand Side [lhs]: The current value of owning a call and a put on the underlying stock over the period of the option at a strike price X is equal to:

Right Hand Side [rhs]: The current value of what value the stock price reaches [FS] in relation to the strike price [X} at the time the option expires.

Alternatively, the put-call equation can be re-written as:

 C + PV[X] = P + PV[FS]

Where PV[FS] = the current spot price of the underlying stock, the [lhs] represents the current value of owning the stock at expiration [long call option plus cash equal to the pv of the strike] sometimes referred to as the fiduciary call, and the [rhs] is the current value of a put against a decline in the underlying below the strike X plus the spot value of the stock [protective put].

Both sides to this equation have a payoff of MAX[S(t), X] at expiration, either the stock price is above the strike in which case the payoff is S(t) or it ends below the strike and the payoff is X.

Derivation [based on arbitrage]

Assume the put and call options are European options that trade in a liquid market where there are no transactions costs for buying or selling. In addition, the underlying stock also trades in an efficient, liquid market free of transactions costs. Under the circumstances there are no arbitrage possibilities [an investor may not gain from price differences in two markets because there are no advantages once storage and carrying costs are taken into account]. Based on the concept of rational pricing, two portfolios that have the same payoff at time T must have the same value at any time prior to time T. If not one could perform arbitrage by going long in the cheaper portfolio and simultaneously shorting the other to make a riskless profit. However, this result violates the initial assumption that there are no arbitrage possibilities. Further, the process of buying the long portfolio would increase its price, while selling the short portfolio would lower its price leading to an equilibrium that eliminates market inefficiency.

Proof [no dividends]: Consider a call and put option with identical strike X expiring at terminal time T on a stock S that pays no dividend. Assume you can find a bond that pays $1 at maturity T. Let S(t) represent the price of the stock at time t. You create portfolio 1 by buying a call option C, and selling a put P with a strike of X and maturity [expiration] T. The payoff for this portfolio is S(T) -X. Alternatively, you assemble portfolio 2 by purchasing one share of stock a borrowing X bonds. In this latter instance, the payoff is again S(T) - X, because the share bought for S(t) will be worth S(T) at expiration, and the borrowed bonds will cost -X at the same time. Portfolios 1 and 2 have the same payoffs and are equivalent to each other at time T. However, if the two portfolios are equivalent at terminal time T, they must also be the same at any time prior to T [i.e. t].

Therefore,

 C(t) - P(t) = S(t) - PV[X]

where S(t) = PV[FS] the current spot price of the underlying stock. Consequently, given the absence of arbitrage, the above relationship [known as the put-call parity equation] holds that for any three prices call, put, bond or stock, one can compute the implied price of the fourth.

Put-call Parity with Dividends: With dividends the derivation is modified in that portfolio 1 consists of going long a call, short a put, and Div(T) bonds that pay $1 at time T [these bonds will be worth Div(t) at time t]. Portfolio 2 remains the same. At time T, the stock will be worth not only S(T), but also the value of the dividends Div(T), which leads to a modified put-call equation of:

 C(t) - P(t) + PV[Div(t)] = S(t) - PV[X].

**Example 1—Verifying the Put-Call Parity with Real Prices**

 On March 2, 2015, financial data on Intel (INTC) is as follows:

Stock Price = $33.25

Option Strike: $35, expiration April 2, 2015 [30 days out]

INTC Put Bid/Ask = $1.88/$1.99 = Average price of $1.94

INTC Call Bid/Ask = $.21/$.25 = Average price of $.23

Lowest Margin Interest rate (that I could find): 4.3%

Substituting the above numbers into the put-call parity equation and using the average prices of the put and call, and using 1/12 of a year = 1 month, we get:

$.23 + $35\*(1.043)^1/12 = $33.25 + $1.94

$.23 + $35 \* 1.003515 = $ 35.35 [lhs]

$33.25 + $1.94 = $35.19 [rhs]

$35.35 is approx. equal to $35.19

The difference between these two figures is $.16 well within an arbitrageur’s trading costs.

**Example - 2 Conversion [Cash and Carry] Arbitrage - Profiting from an Overpriced Call**

Let us suppose that, in the above example, the call was selling for $.80 instead of $0.23. We can then profit from what is known as **conversion arbitrage**. We sell the left hand of the equation and buy the right for an immediate profit. Note that the profit is the same regardless of where the stock price is on expiration — i.e., a **risk-free arbitrage opportunity**.

Conversion Arbitrage Profits per Share

|  |  |  |  |
| --- | --- | --- | --- |
|  **Position** | **ImmediatePayoff** | **Cash Flowin 2 months** |  |
| **St** = Stock Price at expiration. |  | **St ≤ 35** | St > 35 |
| Borrow $35/(1.043)1/12 = 34.88 | +34.88 | -35 | -35 |
| Buy Stock | -33.25 | St | St |
| Write (sell) call | +.80 | 0 | 35 - St |
| Buy put | -1.94 | 35 - St | 0 |
| Total=  | $.49/share | 0 | 0 |

Trading Strategy: Borrow $35 at a margin interest rate of 4.3% per annum, buy the stock with the loan proceeds, write a covered call and buy a put at a strike price of $35 due to expire on month from now on April 2, 2015.

**Example 3—Reverse Conversion [Cash and Carry] Arbitrage - Profiting from an Overpriced Put**

Now suppose that the put is priced at $2.35 instead of $1.94 so that it's overpriced. The other variables remain the same. This arbitrage is called a **reverse conversion**, because it is basically the reverse of a conversion. Now we want to buy the left side of the put-call parity equation and sell the right side. Note that the call covers the shorted stock if the stock rises above the strike, and the put is covered by the shorted stock, if the stock price is less than the strike, which explains St - 35. For instance, if the stock price is $30 on expiration day, the liability from the put is $5 which is mostly covered by the profit of $3.25 from the shorted stock. In this case, the gain from the shorted stock $3.25 plus the put premium credit of $2.35 turns out to be $5.60 which more than covers the $5 loss on the put at expiration and the $.23 for the long call option.

Reverse Conversion Arbitrage Profits per Share

|  |  |  |  |
| --- | --- | --- | --- |
|  **Position** | **ImmediatePayoff** | **Cash Flowin 2 months** |  |
| **St** = Stock Price at expiration. |  | **St ≤ 35** | St > 35 |
| Buy Discounted Note for PV(35) | - 34.88 | + 35 | + 35 |
| Buy call | - 0.23 | 0 | St - 35 |
| Write (sell) put | + 2.35 | St - 35 | 0 |
| Total=  | $0.49/share | 0 | 0 |

Trading strategy: short the stock, buy a long call, then purchase a discounted bond that will pay the strike at expiration, and sell [write] a put. This strategy produces a $.49 per share riskless gain over the one month life of the contract.